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The Cox & Shaw factor.

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Although the need for accurate intensity measurements can probably be met in full only by the use of counter techniques, it is unlikely that photographic recording of X-ray diffraction patterns will be superseded in the near future. It is therefore desirable to increase the accuracy of photographic intensity measurements by improvements in both experimental technique and in the interpretation of the data obtained. In the latter connection it is necessary to take account of the angle of incidence of the X-ray beam on the recording film and to make an adequate correction for its effect. This correction has been treated by Cox & Shaw (1930) who give the formula

$$\frac{\sec\psi\{1+e^{-\mu t\sec\psi}(1-C\sec\psi)\}}{1+e^{-\mu t}(1-C)} \qquad (1)$$

as the ratio of the integrated photographic densities recorded on duplitized film by two identical beams respectively incident on the film at an angle ψ to the normal, and along the normal. In this expression μ, t are the absorption coefficient and thickness of the film base and C is the fractional loss of intensity of the beam in traversing one emulsion.

Cox & Shaw obtained experimental measurements which supported the validity of this expression, but their derivation depends on an approximation which can be valid only for a moderate degree of obliquity and a sufficiently thin emulsion. It is found that the error of this approximation can be significant when the thicker emulsions of the present day are exposed at rather oblique angles, and a more precise formula is therefore proposed.

Consider a single photographic emulsion exposed to X-rays so that the exposure is kept within the range of the linear relationship with photographic density. We further assume that the intensity which is photographically effective is proportional to the energy absorbed in the emulsion. This assumption is clearly justifiable if the vehicle of the emulsion has a negligible absorption. If the grains of the emulsion are isotropic in shape, or if anisotropic in shape are randomly oriented, it can be shown by an elementary analysis that this proportionality is not destroyed if the vehicle of the emulsion has a finite absorption coefficient, and the assumption therefore seems to be justifiable in practice. Hence, if

- D is the density produced,
- E is the exposure,
- μ_e is the effective absorption coefficient of the emulsion,
- t_e is the path length in the emulsion

and k is a proportionality constant,

$$D = kE(1-e^{-\mu_e t_e})\,. \qquad (2)$$

The second emulsion receives an exposure diminished by the absorption in the emulsion and in the film base. Let the latter have an absorption coefficient μ_b and let the path length within it be t_b. Then the energy absorbed in the grains of the second emulsion will be

$$k'Ee^{-(\mu_e t_e+\mu_b t_b)}(1-e^{-\mu_e t_e})\,. \qquad (3)$$

Therefore, the total photographic density in both emulsions will be proportional to

$$k'E(1-e^{-\mu_e t_e})(1+e^{-(\mu_e t_e+\mu_b t_b)})\,. \qquad (4)$$

Hence a beam of given intensity incident on the film at an angle ψ to the normal will give a density enhanced with respect to that produced by a similar beam incident normally by a factor

$$\frac{(1-e^{-A\sec\psi})(1+e^{-(A+B)\sec\psi})}{(1-e^{-A})(1+e^{-(A+B)})}\,, \qquad (5)$$

where e^{-A}, e^{-B} are the absorption factors for a normal beam in a single emulsion and in the film base respectively.

Comparing this result with that given by Cox & Shaw (1) we observe that we must identify

$$e^{-A} = 1-C$$

and

$$B = \mu t\,,$$

and then (5) becomes

$$\frac{(1-(1-C)^{\sec\psi})(1+(1-C)^{\sec\psi}e^{-\mu t\sec\psi})}{C(1+(1-C)e^{-\mu t})}\,, \qquad (6)$$

which reduces to (1) if we make the approximation

$$(1-C)^{\sec\psi} = 1-C\sec\psi\,.$$

This approximation will be valid only if both C and ψ are sufficiently small, that is providing the emulsion does not absorb too strongly and the inclination of the beam to the normal is not too great.

Cox & Shaw found, for a particular film, $C = 0{\cdot}22$ for Cu $K\alpha$ radiation. In this case the enhancement at an inclination of 50° to the normal (a reasonable maximum) is

1·41 (Cox & Shaw formula) and 1·35 (formula (6)).

Such a maximum error of between 4 and 5% is probably tolerable for many purposes, but it has been found that a modern film, which no doubt has a thicker emulsion, has $C = 0{\cdot}35$. In this case the two formulae give enhancements of 1·36 and 1·26 respectively, a difference which is less acceptable. This discrepancy will of course be even larger when longer-wavelength radiation is used.

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Reference

Cox, E. G. & Shaw, W. F. B. (1930). *Proc. Roy. Soc.* A, **127**, 71.